The Conditional Mean Spectrum Based on Eta Indicator

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ABSTRACT

Ground motion record (GMR) selection is an important issue in the structural seismic response-history analysis. For this purpose a set of GMRs with specific properties should be selected. The code-based approaches propose to select a set of records which shall be compatible with a target spectrum e.g. Uniform Hazard Spectrum (UHS). Many research results have shown that using the UHS leads to significant bias in the structural response assessment. Recently the Conditional Mean Spectrum (CMS) has been proposed by Baker et al. which uses the epsilon indicator advantages. Consideration of the correlation of the response spectrum values is the most important feature which was employed in the CMS concept. The epsilon based CMS can reduce the bias in the estimation of the structural seismic response. On the other hand, a new indicator, named eta, has been proposed recently as a linear combination of the conventional epsilon and Peak Ground Velocity (PGV) epsilon, which shows more efficiency and robustness than the conventional epsilon (Mousavi et al. 2010). The eta based CMS (ECMS) has been introduced in this paper. The results based on ECMS spectrum show that the structural response can be predicted with less bias in comparison with using the conventional CMS spectrum.

Keywords: Uniform Hazard Spectrum (UHS), Conditional Mean Spectrum (CMS), Epsilon indicator, Eta indicator, Ground motion record selection.

1. INTRODUCTION

Nonlinear response-history analysis is becoming more common in seismic analysis and design of structures. Most of the seismic design codes and guidelines require selecting ground motion records in which they match the design spectrum within a period range of interest. Careful ground motion selection can reduce the bias and variance of structural response and it can be achieved by using more advanced Intensity Measures (IMs) of ground motion records e.g. using spectral acceleration at the first period of structure and a given damping ratio. Guidelines and building codes introduce a design spectrum which is usually taken as the Uniform Hazard Spectrum (UHS) [1].

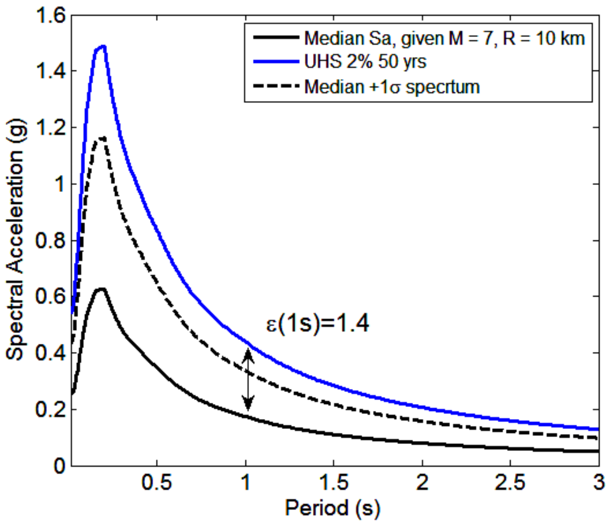

Figure 1: *Median predicted response spectrum given M=7 & R=10 km, and UHS 2% 50 years for an ideal site (using CB08 attenuation relationship).*

The UHS is an elastic spectrum defined on the basis of the seismic hazard analysis at the site where the structure is supposed to be located. But many research results have shown that using the UHS leads to significant bias for the structural response [2]. The shape of UHS for a given ground motion hazard level can be quite different from the shape of the median response spectrum based on an attenuation relationship as seen in Figure 1.

Spectral shape characteristics are especially important for spectral collapse assessments since it is at high amplitude that these differences are most significant [3]. Therefore the choice of ground motion records largely impacts the collapse assessment when assessing the probability of collapse under high amplitude motions [2]. Recently the ground motion parameter epsilon has been used as an indicator of spectral shape by Baker et al. [4]. The epsilon is a measure of the difference between the spectral acceleration of a record and the mean of a ground motion prediction equation at a given period. The epsilon can be defined as Equation (1)

$$\varepsilon_{Sa(T)} = \frac{\ln Sa(T) - \mu_{\ln Sa}(M, R, T, \theta)}{\sigma_{\ln Sa(T)}} \qquad (1)$$

where $\mu_{\ln Sa}(M,R,T,\theta)$ and $\sigma_{\ln Sa(T)}$ are the predicted mean and standard deviation, respectively, of lnSa at a given period, and lnSa(T) is the natural logarithm of the spectral acceleration of interest. The predicted mean is a function of the period of interest (T), the earthquake magnitude (M), distance (R) and local site conditions and faulting mechanism (θ). Taking the epsilon values into account for ground motion selection is important as it can reduce the bias in the estimation of the structural seismic response. New target spectrum called Conditional Mean Spectrum (CMS), which has been employed the epsilon advantages, showed that it can be a suitable tool for the reliable selection of ground motion records [5]. The proposed approach accounts for the magnitude, distance and epsilon values likely to cause a given target ground motion intensity at a given site. On the other hand, the epsilon indicator is investigated more precisely and an alternative indicator of spectral shape, named eta, was proposed by Mousavi et al. which results in more reliable prediction of the non-linear response of structures [6]. The new eta indicator is a linear combination of spectral acceleration epsilon and the peak ground velocity epsilon. It was shown that the record selection based on target eta is more reliable than the selection based on target epsilon [6]. For the above reasons, an effort has been done to find a new target spectrum with eta advantages. In the current study the conditional mean spectrum based on eta indicator, named ECMS hereafter, has been introduced. Using the GMR selection based on ECMS can be more reliable in estimation and assessment of the spectral response.

2. A BRIEF HISTORY ON EPSILON AND ETA INDICATORS

Recent studies have shown that for ground motion records with the same spectrum value at a given period, the spectral shape has an important influence on the response of higher modes of structures as well as on its non-linear behavior [3]. It was shown that the epsilon indicator can be a robust predictor of spectral shape as well as the structural non-linear response [3]. In particular, for a given ground motion hazard level (e.g. 2% chance of exceedance in 50 years), the shape of UHS can be quite different from the shape of the median predicted spectrum for a causal event. This is the reason why the epsilon indicator has been introduced. Meanwhile, it was shown that epsilon has high correlation with structural collapse capacity values [3]. So these summarized advantages are enough to identify epsilon as an applicable indicator in structural analysis and design. Current epsilon is based on only one intensity measure (Sa), but Mousavi et al. have recently shown that a simple combination of IM epsilons can result in more robust prediction of the spectral shape. In other words, the peak ground velocity epsilon associated with conventional Sa epsilon is more effective than other IM epsilons. A linear combination of these two important IM epsilons was introduced as a new indicator of elastic spectral shape and this new indicator, named eta, has shown more correlation with non-linear response. In fact the eta has improved the correlation with collapse capacity by approximately 50 percent. The eta indicator can be defined as Equation (2)

$$\eta = 0.472 + 2.730\varepsilon_{Sa} - 2.247\varepsilon_{PGV} \qquad (2)$$

where ε_{Sa} and ε_{PGV} are respectively, the observed spectral acceleration epsilon and peak ground velocity epsilon.

3. ETA-BASED CONDITIONAL MEAN SPECTRUM

The aim of the current research is to introduce the eta-based conditional mean spectrum as a new target spectrum for the record selection purposes. First it is needed to define a target spectral acceleration value at a period of interest. The period of interest can be computed by modal analysis for a particular structure. Usually the target period is chosen equal to the first mode period of vibration. The mean causal magnitude (M), the mean causal distance (R) and the mean causal epsilon can be obtained by disaggregation analysis based on the probabilistic seismic hazard analysis (PSHA) [7]. The mean predicted spectral acceleration and the corresponding standard deviation of logarithmic spectral acceleration can be computed using existing ground motion prediction models e.g. CB08 which was used in current study. Now the CMS value at the target period can be calculated easily by using Equation (1). The probability calculation shows that the epsilons at other periods are equal to the original epsilon value multiply by the correlation coefficient between two epsilons. The correlation coefficient can be obtained by Baker’s prediction equation as a close form solution [8], or using the correlation based on a suitable subset of GMRs (e.g. from NGA database). GMRs used in this study were given in reference [9].

For the conditional computation we need the target epsilon as well as the target eta, but the disaggregation analysis only provides the target epsilon. For this purpose the target eta values are normalized to the target epsilon values in Equation (2) [5]. The target eta can now be considered to be equal to the target epsilon which is one of the disaggregation results. The target peak ground velocity (ε_{PGV}) can be obtained as written in Equation (3) by using Equation (2).

$$\varepsilon_{PGV} = \frac{1}{2.247}(0.472 + 1.730\varepsilon_{Sa}^{target}) \qquad (3)$$

Substituting Equation (1) and (3) into Equation (2) can produce the conditional mean spectrum based on eta indicator as written in Equation (4).

$$Sa(T) = \exp(\mu_{\ln Sa} + \frac{\eta^{target}\sigma_{\ln Sa(T)}(\rho_{(\eta(T),\eta(T^*))} + 1.730)}{2.730}) \qquad (4)$$

It is clear that the target Sa value in ECMS is equal to CMS value in target period, replacing ρ value by one. In current study both CMS and ECMS were calculated and the effect of new eta indicator was investigated more in the following simple example.

4. A SIMPLE EXAMPE FOR ECMS SPECTRUM

A simple structure with a first-mode period of 1 second was assumed, and 2% probability in 50 years was considered as a given hazard level. Shear wave velocity and other seismic parameters are given as:

- Shear wave velocity = 760 (m/s).
- Depth to the top of co-seismic rupture = 0 (km).
- Rake angle = 35 (degree).
- Dip = 90 (degree).
- Depth to the 2.5km/s shear wave velocity horizon = 2.5 (km).

The mean predicted spectral acceleration equal to 0.17g and the standard deviation equal to 0.66 in the target period (1sec) were obtained by using CB 08 attenuation model [10]. The mean causal values from disaggregation analysis are required. Therefore the following mean values were assumed for an ideal site:

- Mean causal magnitude: 7.0
- Mean causal distance: 10 km
- Mean causal epsilon: 1.4

As the obtained epsilon from disaggregation is assumed to be equal to the target epsilon, the other epsilon values at other periods can be determined as well. For this purpose a linear regression (a correlation model) can be employed. Baker and Jayaram proposed a model for correlation coefficients calculation between two epsilon values based on the Chiou and Youngs model [11]. This method with high level of accuracy is consistent enough with other ground motion prediction models. In the current study all parameters, epsilon values, eta values and correlation coefficients are computed based on considered GMR database without using any close form solution. Figure (2) shows a contour of the correlation coefficient between each two arbitrary epsilon or eta values respectively. The period range is taken from 0.01 to 5 sec.

The epsilon and eta values at other periods can be calculated easily by multiplying target value by the corresponding correlation coefficient value which can be summarized in Equations (6) and (7). For comparison of two correlation coefficients obtained by eta and epsilon values, a new correlation equation is defined as Equation (8). This correlation equation expresses the only difference between CMS and ECMS equations. In fact the parameter ρ´ plays the same role as ρ in CMS equation. So for a better comparison the parameter ρ´ is defined.

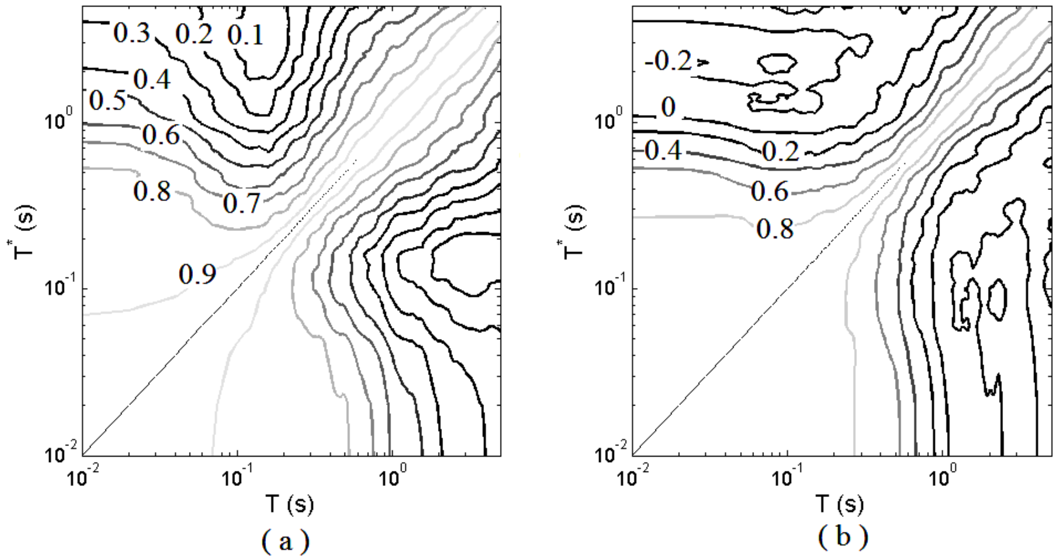

Figure 2: *Empirical correlation coefficients. (a) For epsilon. (b) For eta. (T: Period of interest, T*: Target period)*

$$\varepsilon(T) = \varepsilon^{target} \times \rho(\varepsilon(T), \varepsilon(T^*)) \qquad (5)$$

$$\eta(T) = \eta^{target} \times \rho(\eta(T), \eta(T^*)) \qquad (6)$$

$$\rho'_{(\eta(T),\eta(T^*))} = \frac{\rho_{(\eta(T),\eta(T^*))} + 1.73}{2.730} \qquad (7)$$

Finally the epsilon-based conditional mean spectrum can be computed based on [4] and the eta-based conditional mean spectrum can be obtained by using Equation (4). Figure (3) shows simple cases of these two important target spectra.

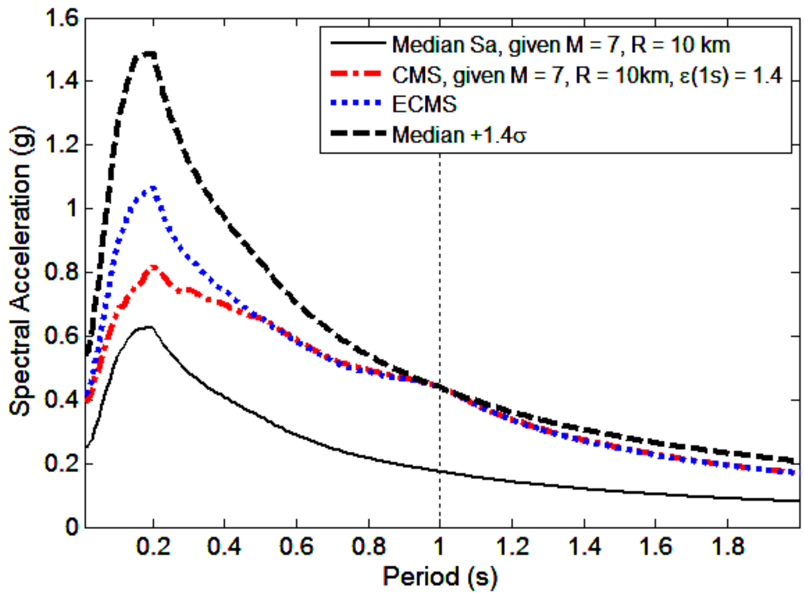

Figure 3: *Epsilon-based and Eta-based conditional mean spectrum for an ideal site, given M=7, R=10 km, ε=1.4*

A brief notice on these curves can detect a very important fact. In the upper period bound which is an essential part in non-linear response of structure, CMS and ECMS are matched well, so the non-linear response seems to be as effective as both CMS and ECMS. It is worth mentioning that an important difference between CMS and ECMS is apparent in lower period range which can influence the higher modes of structures. Both CMS and ECMS have a peak near period of 1 second since the correlation coefficient is high near the target period. Correlation coefficients decrease at large and small periods but the reduction process is more in CMS from target period to smaller periods. In other words, ECMS values in smaller periods are more than the CMS values. This fact is also shown in Figure (4) where the parameter ρ´ for eta and ρ for both epsilon and eta are compared. Note that Figure (4) is explaining the correlation values, and do not reflect the spectral acceleration terms, but this figure can justify the differences between CMS and ECMS since CMS is based on ρ and ECMS is based on ρ´. Figure (3) shows that the difference between two important spectra is beginning from approximately period of 0.5 sec to lower periods where this difference is going to start in Figure (4) too. It can be seen in Figure (4) that the eta correlation values are lower than epsilon correlation values. The lower period bound is related to response of higher modes of vibration, so as an important result the CMS is a non-conservative target spectrum against ECMS for short period structures as well as medium period structures with strong higher modes effect.

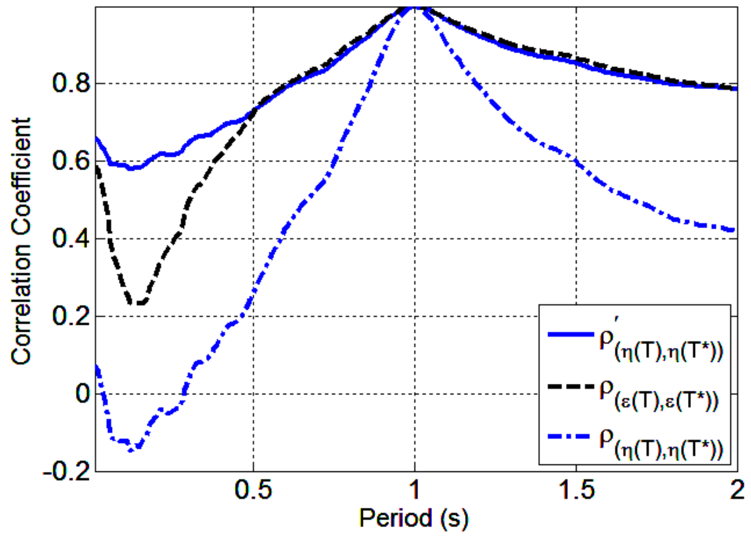

Figure 4: *The correlation coefficients over a period range.*

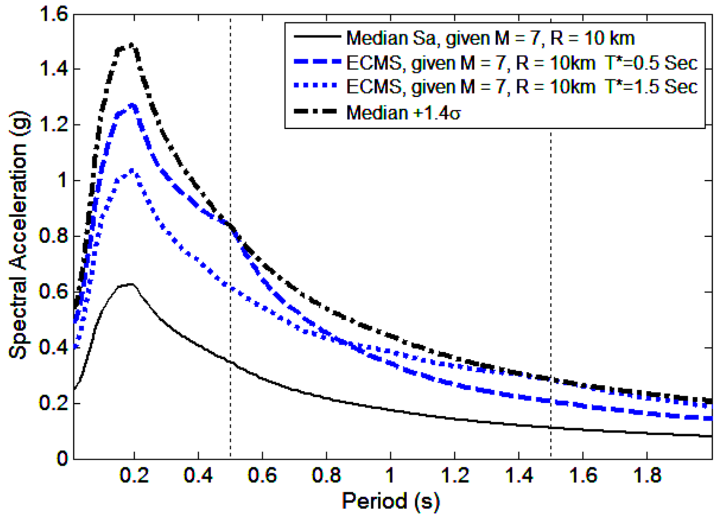

Figure 5: *Eta-based Conditional mean Spectra in different periods for an equal probability of exceedance 2% in 50 years. (T*=0.5 sec & T*=1.5 sec).*

Since the correlation between eta and the structural response is higher than the corresponding correlation between epsilon and the structural response, it can be tentatively claimed that the ECMS is more reliable spectrum than the conventional CMS spectrum. Here the target period is taken equal to the first-mode period of vibration, but this choice cannot be a reliable target, because the sensitivity of the structure is not discussed. So an effort should be made to find the critical target period. For this propose record selection should be done with different CMS or ECMS. Separate sets of selected records based on different CMS or ECMS should be used for analysis and the effect of choosing the target period should be investigated more precisely. Finally it can be inferred that which target period is more sensitive, and it can be chosen as an appropriate target period. Figure (5) shows two ECMS cases computed by different target periods.

5. CONCLUSIONS

For the GMRs selection in non-linear dynamic analysis, different target spectra have been introduced by researchers. The UHS shows that it can be a reliable target but is not suitable because of its conservatism. A new target spectrum, named ECMS, have introduced in this paper which is based on the eta indicator. The ECMS leads to reduction of the bias in the estimation of the structural seismic response since the correlation of eta and the structural response is greater than the correlation between the conventional epsilon and the structural response. It is shown that the ECMS amplitude is greater than the CMS, in short period range, which means that the conventional CMS can underestimate the structural response. Some problems are still remaining for further research e.g. the effect of ECMS based record selection on response of MDOF structures.

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