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REDUCTION OF GROUND MOTION RECORDS FOR SUFFICIENT IDA ANALYSIS

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ABSTRACT

A methodology has been proposed which can be used to reduce the number of ground motion records needed for the accurate prediction of the summarized seismic response (16%, 50% and 84% fractiles) of structures by means of incremental dynamic analysis (IDA). The reduction in the number of ground motion records for accurate prediction of the summarized IDA curves is achieved by introducing a precedence list of ground motion records. The determination of such a list is an optimization problem, which is solved in the paper by means of a genetic algorithm technique. The seismic response of a simple, computationally non-demanding structural model has been used as input data for the optimization problem. The presented example is a three-storey reinforced concrete building, subjected to a set of twenty-four ground motion records. It is shown that the summarized IDA curves can be predicted with an acceptable accuracy by employing only four ground motion records instead of the twenty four, which is the total number of ground motion records for the predefined set.

Introduction

Performance-based earthquake engineering (PBEE) enables quantifying the seismic risk based on a probabilistic approach. A widely used method for PBEE was proposed at the Pacific Earthquake Engineering Research Center (PEER Center). The method decomposes the seismic risk assessment in to four steps: hazard analysis, structural analysis, damage assessment and loss estimation. The structural analysis, which is important within the seismic risk assessment, is usually performed by the Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell 2002) which is a general parametric analysis method for the estimation of seismic demand and capacity for the different levels of seismic intensity measure (IM), based on the ground motion records within a set defining the earthquake scenario. Such an approach requires a huge computational effort, especially due to the many ground motion records involved in the analysis. In order to reduce the computational effort, a number of different approximate methods have recently emerged. Most practical, i.e. approximate methods for IDA analysis involved the replacement of nonlinear dynamic analysis by a combination of pushover analysis of a multi-degree-of-freedom (MDOF) model, and nonlinear dynamic analysis of a single-degree-of-freedom (SDOF) model (Vamvatsikos and Cornell 2005, 2006), (Han and Chopra 2006), and (Dolšek and Fajfar 2005).

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On the other hand a methodology has been proposed with the mean of reducing the number of ground motion records needed for sufficiently accurate prediction of a median IDA curve (Azarbakht and Dolšek 2006). In this paper an attempt has been made in order to extend the proposed methodology to be applicable for prediction of the summarized IDA curves (16%, 50% and 84% fractiles). For this purpose a precedence list of ground motion records has been introduced, which can be determined by utilizing a simple model (e.g. SDOF model) in combination with an optimization procedure. The methodology is illustrated using an example of a three-storey reinforced concrete building subjected to a free -field set of twenty -four ground motion records.

Methodology

The aim of the methodology is to decrease the number of ground motion records needed for the prediction of the summarized IDA curves (16%, 50% and 84% fractiles). In addition to the MDOF model, which is employed in the IDA analysis (Vamvatsikos and Cornell 2002), the advantages of the simple model (e.g. the SDOF model), which is not computationally demanding, are taken into account. Such an approach is employed in other approximate methods (e.g. (Vamvatsikos and Cornell 2005) and (Han and Chopra 2006)). These methods use the response of the simple model, in combination with the pushover analysis, to predict the seismic response of the MDOF model. However, the methodology described here employs the simple model only to predict the precedence list of ground motion records. Single-record IDA curves are then calculated, step-by-step using the MDOF model from the precedence list of ground motion records until acceptable tolerance for the summarized IDA curves is reached. The main steps of the methodology are presented in Fig. 1, and can be described as follows:

1. Select a set of ground motion records based on the earthquake scenario. This is the same step as in an IDA analysis. The number of records within the given set can, if so desired, be high, since, when using the methodology, there is no need to compute the seismic response of the MDOF model for all records in order to obtain a good prediction of the summarized IDA curves.
2. Create a MDOF mathematical model which can be used for the simulation of the realistic seismic response of the structure under investigation.
3. Define a simple mathematical model, e.g. a SDOF model. This model should be a good representative of the linear and nonlinear characteristics of the MDOF mathematical model, yet simple enough for it to be possible to perform a large number of non-linear time history analyses, without the need for very time-consuming calculations.
4. Compute single-record IDA curves for the simple model, for all the ground motion records within the given set. Because of the simplicity of the chosen simple model, this should not be a time-consuming task.
5. Based on the results obtained in step 4, arrange the ground motion records within the given set in order to obtain a good precedence list. This is an optimization problem, which is explained in the next Section. The objective of the optimization is to minimize the differences between the “original” and the “selected” summarized IDA curves. The “original” summarized IDA curves are obtained from all the single-record IDA curves (step 4), whereas the “selected” summarized IDA curves are obtained only for the first s ground motion records from the precedence list, where s is the number of “selected” ground motion records.
6. Compute a single-record IDA curve for the MDOF model, starting with the first record from the precedence list. After computation of single-record IDA curves for the s^{th} record from the precedence list (where s is a number greater than or equal to three), compute the “selected” summarized IDA curves and compare it with the “selected” summarized IDA curves obtained from the $(s\text{-}1)^{th}$ records.
7. Repeat step 6 until the difference between the “selected” summarized IDA curves, determined for the s^{th} and $(s\text{-}1)^{th}$ records, is less than the acceptable tolerance, and then stop performing the IDA analysis on the MDOF model.
8. The “selected” summarized IDA curves, calculated from the s single-record IDA curves can be used for further seismic performance assessment.

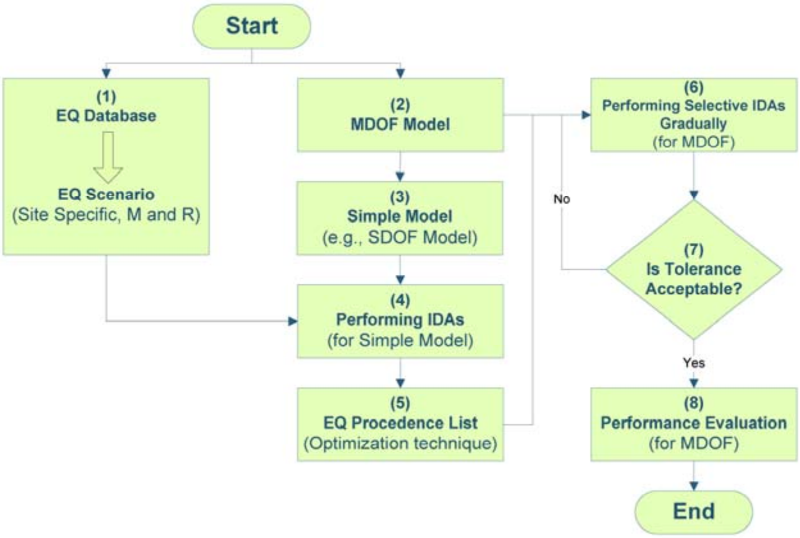

Figure 1. The main steps in the proposed methodology.

The described procedure can significantly reduce the number of nonlinear time history analyses needed to predict the summarized IDA curves with sufficient accuracy. However, the efficiency of the procedure depends on the ability of the simple model to predict the damage measure of the MDOF model, as well as on the ability of the optimization algorithm to find the best precedence list of ground motion records. The summarized IDA curves, obtained from the described procedure by employing a limited number of ground motion records, is usually a good approximation to the "original" summarized IDA curves for the MDOF model, which is calculated from all the single-record IDA curves.

Note that the procedure can be easily applied to other problems, and not just to the problem of minimizing the number of records for the sufficiently accurate prediction of the summarized IDA curves. For example, the procedure can be applied for the selection of a certain number of records for a particular design purpose, since many codes recommend using a certain number of records for the prediction of the most critical actions and/or a different number of records (usually more) for the prediction of the mean or summarized response. In this case the described approach can significantly reduce bias in the seismic response which is present because of the limited number of ground motion records prescribed for nonlinear dynamic analyses.

Steps 1 to 5 of the methodology are illustrated in this paper, whereas steps 6, 7 and 8 are not in the scope of this paper. Interested reader can find more details about steps 6, 7 and 8 in (Azarbakht and Dolšek 2006).

Precedence list of ground motion records

The precedence list of ground motion records was determined for the selected set of ground motion records by employing the Genetic Algorithm (GA) technique (Goldberg 1989). The input data for determining the precedence list are "original" summarized IDA curves (16%, 50% and 84% fractile), single-record IDA curves, both determined on the basis of IDA analysis for the SDOF model, and the corresponding ID numbers of the ground motion records (Table 1). The precedence list of the ground motion records is obtained by rearranging the ID numbers of the ground motion records (Table 1) in order to minimize the fitness function Z:

$$Z = \frac{1}{n-2} \sum_{s=3}^{n} V(s) = \frac{1}{n-2} \sum_{s=3}^{n} [\sum_{f=1}^{3} Error(s, f)] \tag{1}$$

The fitness function is defined as summation of the so-called "partial" fitness function $V(s)$ normalized with the n-2 where n is the number of ground motion records in the set. Z can be therefore interpreted as the average "partial" fitness function $V(s)$. The "partial" fitness function, $V(s)$, is defined as the cumulative error for three fractile curves (f = 16%, 50%, 84%), which are the subject of the optimization.

However, minimization of the “partial” fitness function means the selection of these *s* ground motion records, which are the best representatives of the “original” summarized IDA curves (16%, 50% and 84% fractiles) determined on the basis of IDA analysis for the SDOF model. The *Erorr*(*s*,*f*), which is called error function, is defined as the normalized area, which is determined based on the difference between the “original” and “selected” fractile IDA curve, which can be 16%, 50% or 84% fractile. The error function is a function of a particular fractile curve *f* and of the *s* selected ground motion records for which the “selected” fractile IDA curve is determined. Note, as explained in the methodology, that the “original” summarized IDA curves (16%, 50% and 84% fractiles) are obtained from all the single-record IDA curves, whereas the “selected” summarized IDA curves are obtained for just the first *s* ground motion records from the precedence list, where *s* is equal to or greater than 3, since three fractile curves (16%, 50% and 84%) can be predicted at least with three ground motion records.

The normalized area, expressed in percentage, between the “original” and the “selected” summarized IDA curves, can be calculated as:

$$Error(s,f)=100\times\frac{\int_{0}^{DM_{\max}(s,f)}\left|\Delta IM\left(s,f\right)\right|dDM}{\int_{0}^{DM_{\max,or}(f)}IM_{or}\left(f\right)dDM} \qquad (2)$$

where *DM* is the damage measure, *IM* is an intensity measure for the IDA analysis, $\Delta IM\left(s,f\right)$ is the difference in the *IM* corresponding to the “original” and “selected” *f* fractile IDA curve, and $DM_{\max}\left(s,f\right)$ is the maximum *DM*, as presented in Fig. 2. The parameter $\Delta IM\left(s,f\right)$ depends on the *s* ground motion records which are employed to determine the “selected” *f* fractile IDA curve, and also depends on the *DM*, as schematically shown in Fig. 2. The maximum damage measure $DM_{\max}\left(s,f\right)$ is usually defined by the capacity point on the “original” or “selected” summarized IDA curves. This measure also depends on the number of selected ground motion records *s*. The original maximum damage measure $DM_{\max,or}\left(f\right)$ is usually defined by the capacity point on the “original” summarized IDA curves and $IM_{or}\left(f\right)$ is intensity measure of the “original” *f* fractile IDA curves. Different possibilities of the relationship between the “original” and “selected” fractile IDA curves and the explained parameters of Eq. 2, are presented in Fig. 2.

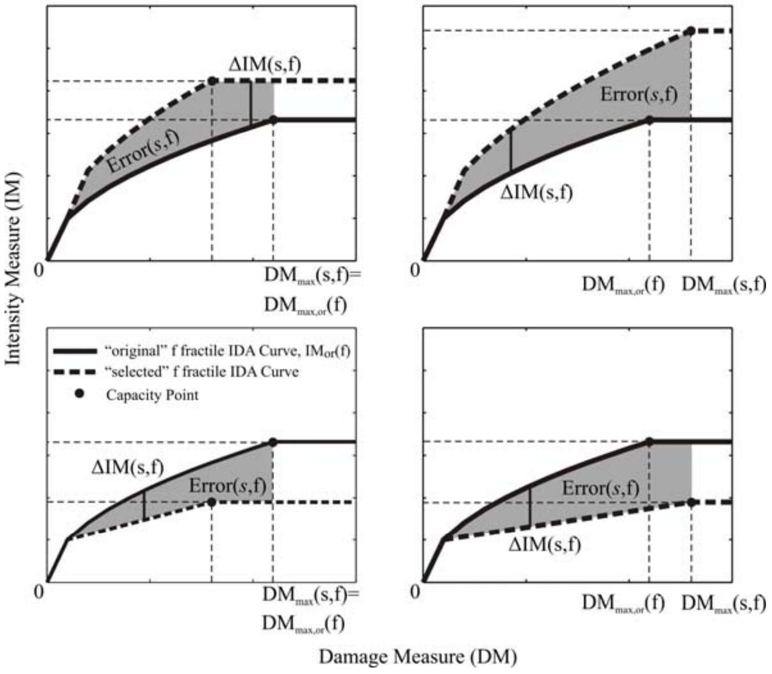

Figure 2. Schematic definition of $DM_{max}(s,f)$ and *Error(s,f)*, shown hatched, based on four possible conditions of the “original” summarized IDA curves and the “selected” fractile IDA curve, which is determined based on *s* selected ground motion records.

Example

In order to demonstrate the applicability of the proposed methodology, a precedence list of ground motion records has been determined for the summarized IDA curves (16%, 50% and 84% fractiles) corresponding to a three-storey reinforced concrete frame building. Precedence list of ground motion records was determined for a set which includes twenty-four ground motion records. The intensity measure selected in the example was the spectral acceleration at the period of the equivalent SDOF model, which was introduced as a good representative of the simple model. The top displacement of the building was chosen as the damage measures. The results are presented in terms of “selected” summarized IDA curves, and compared with the “original” summarized IDA curves.

Ground motion records

A set of twenty -four ground motion records, selected from the PEER Strong Ground Motion Database, was used in the study. The earthquake moment magnitudes M_w for the selected records ranged from 5.5 to 6.5. The selected ground motion records were recorded on NEHRP S_A or S_B, and were uniformly processed by Walter Silva for the PEER Strong Ground Motion Database. The list of records and the corresponding precedence list are presented in Table 1.

Table 1. The free- field set of ground motion records. M_W is the moment magnitude, and R is the closest distance to the fault rupture (given in km).

Event, Year, M_w	ID	Precedence list	Station	R
Morgan Hill, 1984, 6.2	1	12	Gilroy Array #1/G01230	16.2
	2	22	Gilroy Array #1/G01320	16.2
Coyote Lake, 1979, 5.7	3	3	San Juan Bautista/ SJB213	17.9
	4	6	San Juan Bautista/SJB303	17.9
	5	17	SJB Overpass, Bent/SJ3067	19.2
	6	4	SJB Overpass, Bent/SJ3337	19.2
	7	2	SJB Overpass, Bent/SJ5067	19.2
	8	15	SJB Overpass, Bent/SJ5337	19.2
Imperial Valley, 1979, 6.5	9	14	Cerro Prieto/H-CPE147	23.5
	10	5	Cerro Prieto/H-CPE237	23.5
	11	21	Parachute Test Site/H-PTS225	14.0
	12	7	Parachute Test Site/H-PTS315	14.0
	13	13	Superstition Mtn Camera/H-SUP045	26.0
	14	8	Superstition Mtn Camera/H-SUP135	26.0
Livermore, 1980, 5.8	15	11	Antioch - 510 G St/A-ANT270	20.8
	16	20	Antioch - 510 G St/A-ANT360	20.8
	17	23	Fremont - Mission San Jose/A-FRE075	33.1
	18	16	Fremont - Mission San Jose/A-FRE345	33.1
Morgan Hill, 1984, 6.2	19	9	Corralitos/CLS220	22.7
	20	10	Corralitos/CLS310	22.7
	21	18	Gilroy Gavilan Coll/GIL067	16.2
	22	1	Gilroy Gavilan Coll/GIL337	16.2
Parkfield, 1966, 6.1	23	24	Cholame #12/C12050	14.7
	24	19	Cholame #12/C12320	14.7

The test structure and mathematical model

The test structure (referred to in the following as the SPEAR building) is a three-storey asymmetric reinforced concrete frame building, for which a pseudo-dynamic experiment was performed at full scale at the ELSA Laboratory, within the European research project SPEAR (“Seismic performance assessment and rehabilitation of existing buildings”) (Negro and et.al. 2004). This building was designed for gravity loads only. The so-called post-test mathematical model (Fajfar and et.al. 2006) created within the OpenSees program (PEER 1999) was employed for the analyses performed in this study. This mathematical model consists of beam and column elements whose flexural behaviour was modelled by one-component lumped plasticity elements, consisting of an elastic beam and two inelastic rotational hinges (defined by the moment-rotation relationship). A more detailed explanation of the model and a comparison with the experimental results can be found in (Fajfar and et.al. 2006). For reasons of simplicity, the nonlinear dynamic analyses were performed by subjecting the structure to loads in the weak direction only. For this direction the ratio between the base shear and the weight of the building amounted to only about 0.1.

IDA analysis for a simple mathematical model

The simple mathematical model is introduced by a SDOF model, which is based on the results of pushover analysis. Pushover analysis of the MDOF model was performed for the weak direction only, since the mathematical model of the test structure, too, was subjected only to ground motion records in the weak direction. The load pattern employed in the pushover analysis corresponded to the dominant mode shape in the weak direction.

The SDOF model was then defined based on the approach presented in (Fajfar 1999). The force-displacement envelope of the SDOF model was obtained by dividing the forces and displacements of the idealized pushover curve by a transformation factor Γ, which in this example, is equal to 1.26. The period of the SDOF model is 0.92. In order to simulate the hysteretic behaviour of the MDOF model, the same hysteretic rules were used for the SDOF model.

In order to obtain the input data needed to define the precedence list of ground motion records, IDA analysis was performed for the SDOF model by applying the selected ground motion records. The time needed for the IDA analysis performed on the SDOF model is approximately the same as the time needed for one nonlinear time history analysis of the MDOF model.

Results and Discussion

The ID numbers and the precedence list of ground motion records are presented in Table 1. For example, the “selected” summarized IDA curves determined for first four ground motion records from the precedence list are presented in Fig. 3, and compared to the original summarized IDA curves. In Fig. 3a, the comparison is made on the basis of IDA analysis for the SDOF model, while on the Fig. 3b, the results are shown for MDOF model. Very good correlation between the summarized IDA curves is observed, although a few number of ground motion records, four in this case, are used. A good correlation is also, according to the authors observations, present for the selections beyond four.

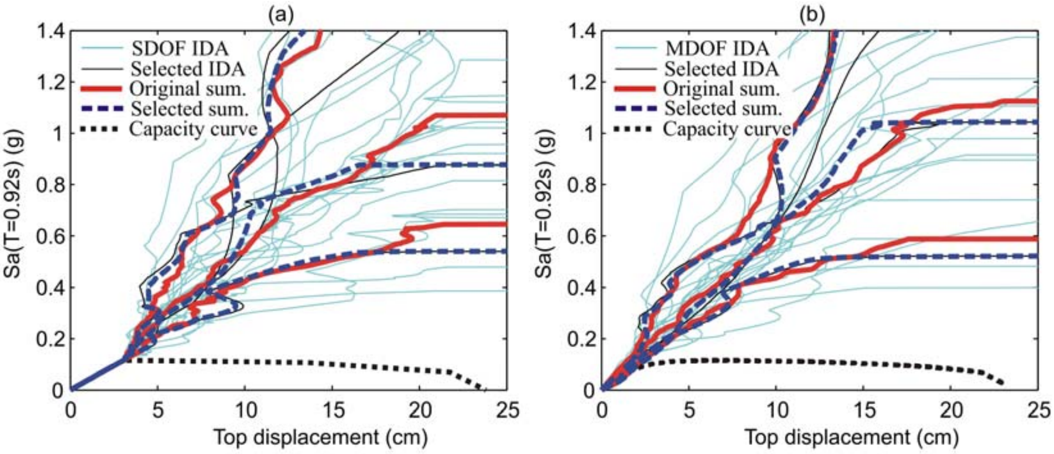

Figure 3. The comparison of “selected” summarized IDA curves using first four ground motion records from the precedence list (Table 1) with the “original” summarized IDA curves: (a) for the SDOF model; and (b) for the MDOF model.

Although for the presented example the four ground motion records are enough for sufficiently accurate prediction of summarized IDA curves question arises, how many selected ground motion records are needed for sufficiently accurate prediction of summarized IDA curves and is it depending on the number of the total ground motion records, which defines the set of records. To investigate this problem a small parametric study was performed for the SDOF model. Additional seventy six ground motion records were added to extend the present set of ground motion records. The total number of records, *n*, is therefore equal to one hundred. Within these ground motion records different set of records were determined, which has a total number of ground motion records 10, 16, 24, 30, 40, 50, 76 and 100. For each set of ground motion records a precedence list of ground motion records was determined.

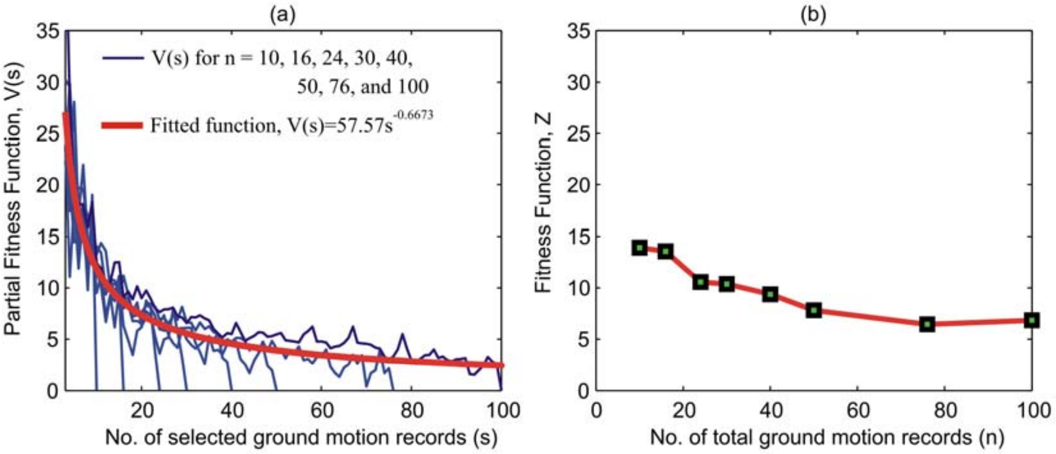

Figure 4. (a) The partial fitness function versus the selected number of ground motion records, *s*, for different total number of ground motion records, *n*, and (b) fitness function versus the total number of ground motion records, *n*.

Although it is clear that the total number of ground motion records is so important for the original summarized IDA curves, an interesting result was observed for the partial fitness function, which, as shown in Fig. 4a, does not depend significantly on the size of the set of ground motion record (*n*). The value of the partial fitness function decreases if the number of selected ground motion records increases, but if the selected number of ground motion record is the same as the total number of records then the value of the partial fitness function is obviously zero (Fig. 4a). The fact that partial fitness function does not depend significantly on the total number of ground motion records means if even total number of records is high then the first four ground motion records may still enough for sufficiently accurate prediction of “original” summarized IDA curves which are on basis of all ground motion records. Even more, based on the results of parametric study, an equation was proposed, valid only for the example presented in this paper, which can be used to predict the partial fitness function $V(s)$ for a given number of selected ground motion records (s) or to predict the number of ground motion records (s) for a given value of partial fitness function:

$$V(s) = 57.6\ s^{-0.69} \qquad or \qquad s(V) = 362.4\ V^{-1.45} \tag{3}$$

For example, for s equals to five, $V(s)$ equals to 22 which means 7% error in average for each fractile. In Fig. 4b, the fitness function Z is presented for different sets of records, which have different total number of records. It can be observed that also fitness function does not significantly change with increasing of n. However the trend can be observed that Z slightly decreases if n increases. This is basically the consequence that there are more choices for selection of ground motion records to predict the summarized IDA curves, if the total number of ground motion records in a set is high. Nevertheless it is possible, as seen in Fig. 4b, to predict the summarized IDA curves with a 10% of average error in each fractile (and for all possibilities of s selected records) with using of about thirty ground motion records as a total number of records in the set.

Conclusions

A methodology has been proposed in order to predict the summarized IDA curves with only a limited number of ground motion records from a given set of records. For this purpose the concept of a precedence list of ground motion records has been introduced. Determination of the precedence list of ground motion records is an optimization problem, which is solved in the paper by a simple genetic algorithm technique. In the proposed methodology, as in other simplified methods, the response of a simple (e.g. SDOF) model is taken into account. Such an approach is not computationally demanding, and can substantially decrease the number of nonlinear dynamic analyses needed for sufficiently accurate prediction of a certain fractile IDA curve.

The methodology was applied to a three-storey reinforced concrete frame building, using a set of twenty-four ground motion records. It was proved that, for this particular example, the 16%, 50% and 84% fractile IDA curves can be predicted with acceptable accuracy by employing only four ground motion records instead of twenty-four, which is the number of all ground motion records in the set of records. The error in the prediction of the summarized IDA curve, in terms of the top displacement is about 5% in average in each fractile. A simple formula is proposed for the accumulated error in the predicted fractile curves versus the desired number of selected records (s).

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