Progressive Incremental Dynamic Analysis for first-mode dominated structures

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ABSTRACT

Incremental dynamic analysis (IDA) is a widely-used method for assessing structural performance under earthquake excitations. It enables direct evaluation of the record-to-record variability in structural response through a set of ground motion records. If the number of ground motion records is large then, the method becomes computationally demanding. To facilitate its practical application, a precedence list of ground motion records has been introduced, aiming at selecting the most representative ground motion records for IDA analysis. In progressive IDA analysis the IDA curves are computed progressively, starting from the first ground motion record in the precedence list. After an acceptable tolerance has been achieved, the analysis is terminated. This approach may significantly reduce the computational effort for first-mode dominated structures, since the seismic response can be computed only for a certain number of ground motion records from the precedence list in order to achieve an acceptable level of confidence in the prediction of the summarized ($16^{th,}$ 50^{th} and 84^{th} fractiles) IDA curves. The proposed implementation of incremental dynamic analysis, which is demonstrated using an example of a four-storey reinforced concrete frame, can also be used for the selection of ground motion records from a very large set of records, provided that all the records properly represent the seismic scenario for a given site.

CE Database subject headings: Ground motion; Earthquakes; Seismic response; Optimization; Building frames; Concrete; Nonlinear analysis.

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INTRODUCTION

Determination of demand and collapse capacity due to earthquakes is an important issue in performance-based earthquake engineering. Many different methods and procedures for assessing seismic structural performance have therefore emerged during the development of performance-based earthquake engineering. For example, methods for assessing structural collapse capacity in order to protect life safety vary from the simplest methods, which can be based on the response of a simple single-degree-of-freedom (SDOF) model, to complex nonlinear dynamic analyses performed for a structural model, which is analyzed for a set of ground motion records (Villaverde 2007). One of the methods commonly used in recent years, is Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell 2002). It involves subjecting a structural model to a number of ground motion records, each scaled to multiple levels of intensities. Although such an approach requires a large number of inelastic time history analyses, it has been used by several researchers for different applications (Zareian and Krawinkler 2007; Liao et al. 2007; Tagawa et al. 2007). Different approximate methods have also emerged, aiming at reducing computational effort. The approximate methods for IDA analysis usually involve the replacement of nonlinear dynamic analysis by a combination of the pushover analysis of a structural model and dynamic analysis of a simple model, e.g. SDOF model (Vamvatsikos and Cornell 2005a; Dolšek and Fajfar 2005; Han and Chopra 2006). However, if it is a requirement that the seismic response of a structure is predicted with the most accurate nonlinear dynamic analysis, then the practical application of incremental dynamic analysis is limited mainly due to the computational effort needed to perform incremental dynamic analysis, but also due to the definition of the seismic loading, which is, in this case, defined by a set of ground motion records. Different questions arise in the process of selecting the ground motion records for the incremental dynamic analysis. Firstly it is important that the selected set of ground motion records reflects the seismic hazard of the particular site and that the scaling of records is “legitimate” (see Luco and Bazzurro, 2007). When these two conditions are not satisfied, a bias in the structural response can occur (Baker and Cornell 2006, Luco and Bazzurro 2007). Note that the scaling of records is “legitimate” if the ratio between the median seismic response parameter to scaled records and the median seismic response parameters to unscaled records (bias) is in the range of the defined tolerable interval. However, careful selection of ground motion records can reduce the bias in the structural response (Shome et. al. 1998; Iervolino and Cornell 2005). On the other hand, many researchers have tried to reduce the dispersion in nonlinear response by introducing the improved intensity measures (Tothong and Luco 2007; Luco and Cornell 2007; Baker and Cornell 2006; Vamvatsikos and Cornell 2005b). The most extensive study for evaluation of ground motion selection and modification methods was recently prepared by PEER Ground Motion and Modification Working Group (Haselton (Ed.) 2009).

The progressive incremental dynamic analysis, which involves a precedence list of ground motion records, and is proposed in this paper, can be used for optimal selection of ground motion records from a given set of records, which can be obtained by employing existing techniques for selection of appropriate set of ground motion records. The precedence list of ground motion records was first introduced by authors for the prediction of the median IDA curve by means of a limited number of ground motion records (Azarbakht and Dolšek 2007). The use of precedence list of ground motion records, introduced in this paper, is extended for the prediction of the summarized IDA curves, i.e. the 16th, 50th and 84th fractiles. The precedence list of ground motion records provides the advantage of a simple mathematical model, which is not computationally demanding, and it is defined as an optimization problem, which is solved by means of a genetic algorithm, also used in seismic engineering for the optimal design of structures (Foley et al. 2007), as well as a proposed simple procedure which will be explained in detail later in the paper. Once the precedence list of ground motion records is known, the IDA curves are computed progressively for the MDOF structure, starting from the first ground motion record in the precedence list. When the required tolerance in the prediction of the summarized IDA curves is achieved, the analysis can be terminated. The proposed method is demonstrated using the example of a four-storey reinforced concrete frame structure subjected to a set of ground motion records, which consists of ninety-eight ground motion records.

PROGRESSIVE INCREMENTAL DYNAMIC ANALYSIS

A new element of progressive IDA, when compared to the elements of the IDA (Vamvatsikos and Cornell 2002), is the precedence list of ground motion records. This difference is shown schematically in Fig. 1. In general, IDA curves are calculated for all the ground motion records in a set of such records (Fig. 1a), while in progressive IDA (Fig. 1b), the IDA curve is first calculated for the first ground motion record from the precedence list, and then progressively for other ground motion records from the precedence list of these records. After calculation of several IDA curves, the analysis can be terminated, since the acceptable tolerance is achieved. It is convenient to check the tolerance only after every three IDA curves are computed, since three different fractile quantities of the response are going to be calculated. However, the tolerance can be evaluated only after the IDA curves are calculated for the second subset of three ground motion records, since the tolerance is defined with respect to the IDA curves of the previous subsets as it is explained later in the paper. Optimization based on the defined subsets of ground motion records is effective because the aim for progressive IDA analysis is to predict three summarized IDA curves with a limited number of ground motion records from a set of ground motion records. The total number of subsets of ground motion records (m) is determined as the downward rounded integer of n/3, where n is the total number of ground motion records in a set of ground motion records.

Basically, the benefit of progressive IDA, in comparison to the IDA, is the reduction of the computational effort. However, determination of the precedence list of ground motion records also requires some computational time, firstly for IDA of the simple model (e.g. SDOF system), which is needed in order to establish a precedence list, and secondly for the optimization of the precedence list of ground motion records. The simple model is usually defined based on the results of pushover analysis, which is performed for the complex, multi-degree-of-freedom (MDOF) model. It is important that the simple model is a good representative of the linear and nonlinear characteristics of the MDOF structural model, yet simple enough for it to be possible to perform a large number of non-linear time history analyses, without the need of going through time-consuming calculations. The appropriate simple models can therefore be a single-degree-of-freedom (SDOF) model, or a model which has one degree of freedom per each storey. However, the computational time for determination of the precedence list of ground motion records is usually less than the computational time for the determination of an IDA curve, which is computed for the MDOF model, especially if the structure is complicated.

Determination of the precedence list of ground motion records is, in fact, an optimization problem, which is explained in the next Section. However, the objective of the optimization is to minimize the differences between the “original” and the “selected” summarized IDA curves calculated based on the simple model. The “original” summarized IDA curves are obtained from all the IDA curves, whereas the “selected” summarized IDA curves are obtained only for the first s subsets of the ground motion records from the precedence list, where s is the number of “selected” subsets of ground motion records from m, which is the number of all subsets of ground motion records.

The significant reduction in computational time is not the only benefit of progressive IDA. It can also be interpreted as a procedure for the selection of ground motion records from a very large set of such records. For example, if there are several hundred ground motion records, it is practically impossible to calculate IDA curves for all of these records. In this case, only the progressive incremental dynamic analysis can be used, since the summarized IDA curves do not change significantly after IDA curves are calculated for a certain number of ground motion record from the precedence list of ground motion records. However, it is assumed that all ground motion records are good representatives of the seismic scenario for the given site, and that the scaling of ground motion records is "legitimate".

PRECEDENCE LIST OF GROUND MOTION RECORDS

The precedence list of ground motion records is determined by rearranging the ID numbers of the ground motion records in order to minimize the difference between the "selected" summarized IDA curves and the "original" summarized IDA curves. The difference between these two types of summarized IDA curves, which were explained in the previous Section, is defined by means of an error function:

$$Error(s,f) = 100 \times \frac{\int_0^{EDP_{\max}(s,f)} \left|\Delta IM\left(s,f\right)\right| dEDP}{\int_0^{EDP_{\max,or}(f)} IM_{or}\left(f\right) dEDP} \tag{1}$$

where s is the number of selected subsets of three ground motion records, EDP is an engineering demand parameter of the simple model, IM is an intensity measure for the IDA analysis, $\Delta IM\left(s,f\right)$ is the difference in the IM corresponding to the "original" and "selected" f-th summarized IDA curve, $IM_{or}\left(f\right)$ is the intensity measure of the "original" f-th summarized IDA curves, $EDP_{\max}\left(s,f\right)$ is the maximum of the engineering demand parameters corresponding to the global dynamic instability (i.e. the capacity point as shown in Fig. 2) of the "selected" and "original" f-th summarized IDA curves, and $EDP_{\max,or}\left(f\right)$ is the engineering demand parameter corresponding to the capacity point of the "original" f-th summarized IDA curve, as presented in Fig. 2a. The parameter $\Delta IM(s, f)$ depends on the s selected subsets of the ground motion records which are employed to determine the "selected" f-th summarized IDA curve. Additionally, $\Delta IM(s, f)$ also depends on the EDP, which, for simplicity, is not denoted but is shown schematically in Fig. 2a. The $EDP_{max}(s, f)$ also depends on the number of selected subsets of ground motion records s, since, the "selected" f-th summarized IDA curve differs for different number of selected subsets of ground motion records.

The function *Error(s,f)* is expressed as a percent, and represents the normalized area between the "original" and "selected" f-th summarized IDA curves. Note, as explained in the previous Section, that the "original" summarized IDA curves (16th, 50th and 84th fractiles) are obtained from all the IDA curves, whereas the "selected" summarized IDA curves are obtained for just the first s subsets of ground motion records from the precedence list. Also, the error calculated according to Eq. (1) is a global measure for the error and it is used for determination of the precedence list of ground motion records. Therefore Eq. (1) can not be used to measure the error between the two engineering demand parameters, which are determined by the IDA and progressive IDA at a given intensity measure. However, according to the authors' observation, the error (Eq.(1)) less than 10% can be defined as an acceptable global error. In this case the difference between engineering demand parameter of the "selected" and "original" IDA curves is small for the wide range of intensity measure.

Different optimization techniques can be used for the determination of the precedence list of ground motion records. Similarly, as in the previous study (Azarbakht and Dolšek 2007) the precedence list of ground motion records was determined by means of a genetic algorithm (GA), and also by a proposed simple procedure. In the case of the GA, the fitness function has to be defined and minimized in order to obtain the best precedence list of ground motion records. The fitness function Z can be simply defined as the average cumulative error function calculated for all subsets of ground motion records and for all summarized IDA curves and is normalized by the total number (m) of subsets of the ground motion records:

$$Z = \frac{1}{m} \sum_{s=1}^{m} \sum_{f=1}^{3} Error(s, f). \quad (2)$$

In the first step the GA randomly generates a finite number of precedence lists of ground motion records, called initial population according to GA terminology, which will be quoted in brackets. Each precedence list (individual) is an 1×n array that represents the ID numbers of all the ground motion records in the specified set of such records. A certain number of the best precedence lists of ground motion records (elites) are directly selected for passing into the next step (generation) without any changes. In each step of GA (new generation), some of the precedence lists of ground motion records (new individuals) are generated by means of a crossover function. This combines the sequences (genes) of the two precedence lists of ground motion records (parents) to form a new precedence list of ground motion records (child). In each next step (new generation), some precedence lists of ground motion records (new individuals) are generated by means of random changes of positions of ID numbers of ground motion records in the precedence list (mutation). This operation is a necessary part of the GA, and prevents it from converging to a local optimum. The GA optimization technique, which is used in this paper, clearly cannot define the exact global minimum of the fitness function Z, but it can usually find a solution near the global minimum. The precedence list of ground motion records might be different in a new run, but the predicted summarized IDA curves do not change significantly, because the solution is always near the global minimum. However, many precedence lists of ground motion records are always generated with the GA before obtaining the optimal solution for the precedence list of ground motion records. More details regarding the GA procedure used in this paper can be found in the previous study (Azarbakht and Dolsek 2007). The generation of many precedence lists of ground motion records can be avoided by employing the proposed simple procedure for determining the precedence list, which is explained in the next Section.

A simple procedure for determining the precedence list of ground motion records

The simple procedure for determining the precedence list of ground motion records is based on the assumption that the best precedence list can be obtained gradually, starting with the first ground motion record (ID number) in the precedence list, which corresponds to the minimum value of errors (Eq.(1)) that are calculated for s=1, f=1 and for all records in the given set of ground motion records. Other records in the precedence list of ground motion records are then determined gradually by using the same assumption as in the first step. This is similar procedure as employed in the determination of the precedence list of ground motion records for prediction of the median IDA curve (Azarbakht and Dolsek 2007). However, prediction of precedence lists of ground motion records in this case requires additional steps since the precedence list of ground motion records is provided for prediction of the three fractile IDA curves. The complete algorithm is explained in the following steps:

1) Calculate the IDA curves for all ground motion records and determine the "original" summarized IDA curves (16^{th}, 50^{th} and 84^{th} fractiles) for the simple model (e.g. a SDOF model).
2) Calculate the error function (Eq.(1)) for s=1 and for each ground motion record from the given set of ground motion records, firstly for f=1, i.e. only for the "original" 16^{th} fractile IDA curve. Clearly there are n results for the error function, where n is the total number of ground motion records in the given set. It is clear that the best ground motion record (ID number) for the prediction of the "original" 16^{th} fractile curve is

that with the corresponding minimum value of the error function (Eq.(1)). This defines the ground motion record, which is the first in the precedence list.

3) Increase *f* by one (*f*=2) in order to find the best ground motion records which have the minimum deviation from the "original" median IDA curve. Obviously there are *n-1* records left to be placed in the precedence list of ground motion records. The error function is therefore calculated only for these ground motions records, which are not yet placed in the precedence list of ground motion records. Again, the minimum value of the determined values of error functions defines the second ground motion in the precedence list.
4) Increase *f* by one (*f*=3) in order to select the best ground motion record for the prediction of the "original" 84th fractile curve. At the end of this step the first subset of ground motion records has been determined, and *n*-3 ground motion records are left to be placed in the precedence list of ground motion records.
5) Increase *s* by 1 and evaluate the error function for the "original" *f*-th summarized IDA curve, i.e., begin with setting *f*=1. There are $n-3(s-1)-(f-1)$ different values for the error function, which is the same number as the number of ground motion records still waiting to be placed in the precedence list. In order to evaluate the error function, the "selected" *f*-th summarized IDA curve is determined as the median value of the IDA curves, which are specifically selected from the ground motion records already placed in the precedence list and the additional ground motion record, which is still a candidate for the next place in the precedence list. For example, if *s*=4, the "selected" 16th fractile curve (*f*=1) is determined as the median of the IDA curves, which corresponds to the ground motion records placed in the 1st, 4th, and 7th places of the precedence list and the additional ground motion record which is arbitrarily selected from the other ground motion records, which are candidates for the precedence list of

ground motion records. Again the minimum value of the error function defines the new ground motion record in the precedence list of ground motion records.

6) Continue with step 5 until all the ground motion records are placed in the precedence list of ground motion records.

Tolerance for the selected summarized IDA curves

Once the precedence list of ground motion records has been determined on the basis of IDA analysis for a simple model, it is important to know how much is the difference between the two "selected" summarized IDA curves, each calculated for the *f*-th fractile firstly on the basis of first *s* subsets of ground motion records from the precedence list, and secondly for the *s*-1 subsets of the ground motion records. This measure is called the tolerance function, and is defined as:

$$Tolerance(s, f) = 100 \times \frac{\displaystyle\int_0^{Max[EDP_{\max}(s,f), EDP_{\max}(s-1,f)]} \left| IM(s, f) - IM(s-1, f) \right| dEDP}{\displaystyle\int_0^{EDP_{\max}(s-1,f)} IM(s-1, f) dEDP} \tag{3}$$

where $IM(s,f)$ and $IM(s-1,f)$ are the values of the intensity measures for the *f*-th fractile IDA curves, which are determined, respectively, on the basis of the first *s* and *s*-1 subsets of ground motion records. These subsets of ground motion records are defined with the precedence list of ground motion records. The additional parameters introduced into Eq. (3) are the engineering demand parameters $EDP_{max}(s,f)$ and $EDP_{max}(s-1,f)$, which correspond to the capacity point of the *f*-th fractile curve determined from first *s* and *s*-1 subsets of ground motion records. The described parameters are presented schematically in Fig. 2b.

The tolerance as introduced in Eq. (3) can generally be calculated for selected summarized IDA curves for the simple model and for the MDOF model. In the first case the tolerance can be calculated only for checking the reduction of tolerance with an increasing number of selected ground motion records from the precedence list. However, the evaluation of tolerance is a key element of the progressive IDA analysis once the IDA curves have been calculated for MDOF model. In this case the tolerance is used for decision-making about the sufficient number of subsets for the prediction of summarized IDA curves for MDOF model.

EXAMPLE

The applicability of the proposed progressive incremental dynamic analysis is demonstrated by determining a precedence list of ground motion records in order to predict the summarized IDA curves (16th, 50th and 84th fractiles) for a four-storey reinforced concrete frame building by employing only a limited number of ground motion records. The precedence list of ground motion records was determined for a set which included ninety-eight ground motion records. The maximum inter-story drift ratio of the building was chosen as the engineering demand parameter. The results are presented in terms of the "selected" summarized IDA curves, and compared with the "original" summarized IDA curves.

The test structure and the mathematical model

The four-storey reinforced concrete structure (Fig. 3) was selected to demonstrate progressive incremental dynamic analysis. For this structure different pseudo-dynamic tests were performed at the European Laboratory for Structural Assessment (ELSA, Ispra) (Negro and Verzeletti 1996; Negro et al. 1996). The structure was designed according to previous versions of Eurocodes 2 and 8 (Fardis (ed.) 1996). The design base shear versus the weight of the structure corresponded to about 16% (Fardis (ed.) 1996).

The same principles of the modeling as those presented by Fajfar et al. (2006) were employed in this study too, basically, the mathematical model of the test structure is developed in compliance with the Eurocode 8 (CEN 2004) requirements. Beam and column flexural behaviour was modeled by one-component lumped plasticity elements, composed of an elastic beam and two inelastic rotational hinges (defined by the moment-rotation relationship). The element formulation was based on the assumption of an inflexion point at the midpoint of the element. For beams, the plastic hinge was used for major axis bending only. Bilinear moment-rotation relationships were used for the moment-rotation relationship. However, the strength degradation was also modeled. The axial forces due to gravity loads were taken into account when determining the moment-rotation relationship for plastic hinges in the columns. These principles are in full compliance with Eurocode 8. All analyses were performed by OpenSees (McKenna et al. 2000). A comparison between the calculated and experimental time histories for the top displacement and base shear can be found in Fajfar and et. al. (2006).

Ground motion records

The event under consideration is a magnitude 7.0 earthquake (magnitudes are between 6.8 to 7.1) occurring on a strike-slip fault, at a site that is 10 km from the fault rupture and having a top 30 m of the soil profile shear-wave velocity of 400 m/s (Haselton (Ed.) 2009). The ninety-eight ground motion records were selected carefully (Haselton (Ed.) 2009) from the PEER Strong Ground Motion Database (PEER 2005). The average epsilon (Baker and Cornell 2006) of un-scaled records, which was calculated according to Campbell-Bozorgnia (2007) attenuation relationship, is equal to 0.95 for the building's first-mode (T_1=0.66 s). For brevity only the acceleration spectra of the selected set of ground motion records are presented in Fig. 4. More details about the selected set of ground motion records can be found in the PEER Report 2009/01 (Haselton (Ed.), 2009).

Definition of simple mathematical models and corresponding IDA curves

The simple mathematical model was defined as the single-degree of freedom (SDOF) model, which had been used in the previous study (Azarbakht and Dolšek 2007). Pushover analysis was first performed for the positive and negative directions of loading (Fig. 5). The load pattern corresponded to the product between the components of the storey mass vector M=[87, 86, 86, 83] t and the first mode shape vector ϕ=[0.296, 0.603, 0.858, 1]. The pushover curves were idealized with a tri-linear force-displacement relationship, as presented in Fig. 5. Symmetric positive and negative backbone curves were assumed. However, idealization of the pushover curves is determined on the basis of engineering judgment, since the results based on the SDOF model are used only to determine the precedence lists of ground motion records, and not for the performance assessment of the structure. The force-displacement envelope of the SDOF model was obtained by dividing the forces and displacements of the idealized pushover curve (Fig. 5) by a transformation factor Γ (Fajfar 2000), which, in this example, is equal to 1.266. The mass of the SDOF model (234.5 t) was calculated as the scalar product of the mass vector and the transposed first mode shape vector. The period of the SDOF model is T^*=0.71 s which is close to the fundamental period (0.66 s) of the MDOF model. The same hysteretic rules as were used in the plastic hinges of the MDOF model were also employed in the case of the SDOF model.

Incremental dynamic analysis (Vamvatsikos and Cornell 2002) for the simple (SDOF) model was then performed and it is not computationally demanding, since the computation time for the determination of the IDA curves of all ninety-eight ground motion records is about the same as the time needed for the determination of one IDA curve for the MDOF model. Note that the intensity measure, which corresponds to global dynamic instability, was determined with the tolerance of 0.02 g. The spectral acceleration $S_a(T^*)$, which corresponds to the period of the SDOF model, was selected as the intensity measure, and also used in the case of the progressive IDA for the MDOF model. The IDA curves for the simple model and the corresponding summarized IDA curves (counted 16^{th}, 50^{th} and 84^{th} fractiles), are presented in Fig. 6.

Precedence lists of ground motion records determined on the basis of IDA analysis for the simple model

The results obtained from the IDA analysis for the SDOF model and the ground motion records IDs are the input data for the determination of the precedence list of ground motion records. The precedence list of ground motion records, in this study, was determined by employing two procedures: (1) a GA based optimization technique, and (2) a simple procedure. Similar procedures were also used in the previous study (Azarbakht and Dolsek 2007). Note that the simple procedure, as described in this paper, focuses on the prediction of the three fractile IDA curves and it is therefore different from that used in the previous study which was focused only on the prediction of the median IDA curve, whereas the GA based optimization technique (Matlab 2004) is the same but involves the new fitness function (Eq. (2)), which was modified to calculate the three fractile IDA curves instead of the median IDA curve. Although only three records are the same in the case of the two different methods, if first fifteen ground motion records are compared from the precedence lists, the "selected" summarized IDA curves are very similar as it will be shown later.

Once the precedence list has been determined, the quality of the solution can be measured by two parameters, $Error(s,f)$ and $Tolerance(s,f)$, which are defined in Eqs. (1) and (3), respectively. Both parameters are presented in Figs. 7 and 8 for the solution obtained in the case of the simple procedure, and for that obtained in the case of the GA optimization technique. In general, both parameters decrease if s increases, where s represents the first s subsets of ground motion records from the precedence list, which are employed for calculation of the "selected" summarized IDA curves. However, in the case of the simple procedure and at high values of s the error and the tolerance tends to increase for some fractile curves since the optimization algorithm based on the simple procedure has the ability for optimization only in the case of low s. In the last stages of determination of precedence list of ground motion records according to the simple procedure there are only few records left and thus there is less chance for selection of records, which increases the error. On the other hand such weakness is not problematic since the objective of the progressive IDA analysis is to use a few subsets of ground motion records from the top of the precedence list.

The precedence list of ground motion records is slightly better optimized if determined by the GA optimization technique. However, the quality of the solution based on the simple procedure is acceptable even if only six ground motion records (two subsets of ground motion records s=2) are selected from the precedence list, as presented in Fig. 9, where the original "summarized" IDA curves are compared with the "selected" (s=2) summarized IDA curves. Practically no differences can be observed between the original "summarized" IDA curves and the "selected" summarized IDA curves.

Progressive IDA analysis for MDOF model

Progressive IDA analysis was performed for both of the precedence lists which were determined with the simple procedure and the GA optimization technique. The intensity measure, which corresponds to global dynamic instability (collapse), was determined with the tolerance of 0.02 g as was used in the case of the IDA analysis for the simple model. The IDA curves for the MDOF model were calculated progressively, starting from the first subset of the ground motion records from the precedence list. Since, the "original" summarized IDA curves for the MDOF model are not known, there is no confidence in the "selected" summarized IDA curves for s=1. For this reason the IDA curves have to be calculated for at least the first six ground motion records from the precedence list (two subsets of ground motion records, s=1 and s=2), which is the minimum number for the calculation of the tolerance (Eq. (3)). If the tolerance is small, then the progressive IDA analysis can be terminated. Usually the acceptable tolerance is less than 10%. In the case of the current example, the tolerance for s=2 (i.e. first six ground motion records) exceeds 10% only for the "selected" 16^{th} fractile curve, but for the precedence list calculated on the basis of the simple procedure only. Although the tolerance is low even for s=2, IDA curves were calculated progressively up to s=5 (15 ground motion records). For s=5 the tolerance, as presented in Fig. 10 is much less than 10%. It was therefore concluded that the "selected" summarized IDA curves calculated on the basis of the first 15 ground motion records from the precedence list represent a high-confidence approximation of the "original" summarized IDA curves, which would be determined on the basis of all 98 ground motion records.

The "selected" summarized IDA curves for both precedence lists are presented in Figs. 11 and 12, respectively, for s=2 and s=4 only. The "selected" summarized IDA curves are compared to the "original" summarized IDA curves. There is very good agreement, as seen in Figs. 11 and 12, between the "selected" and "original" summarized IDA curves, even in the case of s=2 (six ground motion records). Better agreement was also observed in the case of predictions of the summarized IDA curves with more than six ground motion records, e.g. for 12 ground motion records (s=4), as presented in Figs. 11 and 12, respectively, for the precedence list determined on the basis of the GA optimization technique and the simple procedure. Although the precedence lists of ground motion records differ if they are determined, respectively, by the GA optimization technique and the simple procedure, only a minor difference can be observed between the corresponding "selected" summarized IDA curves (Fig. 11 and 12). However, in both cases the collapse capacity is slightly underestimated, especially for the prediction of the 50^{th} fractile.

Discussion of the results

The precedence list of ground motion records is determined on the basis of the simple model, which is only a rough approximation of the MDOF model. It could therefore be expected that the error (Eq. (1)), if calculated for the results based on the MDOF model, would be larger than that for the SDOF model. In order to calculate the error for the MDOF model, which is presented in Fig. 13, IDA curves have to be calculated for all 98 ground motion records. In general, as seen in Figs. 7 and 13, the error determined for the MDOF model is only slightly larger than the error obtained for the SDOF model, if the values are compared for low numbers of selected subsets of ground motion records (s). The same trend can be observed from the tolerance, which is presented in Figs. 8 and 10. The larger error and tolerance in the case of the MDOF model can be mainly attributed to the different collapse capacities between the IDA curves obtained on the basis of the MDOF and the SDOF models. Only one collapse mechanism, which results from the pushover analysis, is simulated in the SDOF model, whereas in the case of the MDOF model collapse may appear in different forms. Thus, more potential collapse mechanisms may reduce the effectiveness of the progressive IDA analysis. Also, other engineering demand parameters, which are not well-correlated with the first-mode roof drift, may not be predicted with the same accuracy as the top displacement or maximum inter-storey drift since the precedence list of ground motion records is determined for the displacement of the SDOF system. However, in the case of the test structure it was shown that the SDOF model is a sufficient representative for the simple model. For different structures, especially for special buildings or bridges, which are not first mode dominant, the SDOF model may not be sufficiently representative for the simple model.

The simple procedure and the GA optimization technique were proven to be acceptable procedures for the determination of the precedence list of ground motion records. The simple procedure may have an advantage since it is very easy to program, produces a unique solution in each run, and is also much faster than the GA optimization technique. However, the "selected" summarized IDA curves obtained from the different precedence lists are more or less the same.

A small parametric study was performed in order to investigate how many randomly selected records from a set of records are needed to predict the $Error(s,f)$ (see Eq.(1)) as effective as it is obtained by the precedence list of ground motion records, which is determined by the GA or simple procedure. For example, the $Error(s,f)$ (see Fig. 14a), computed for randomly determined precedence list of ground motion records, is significantly higher than that presented in Fig. 13. In addition, a probability of $Error(s,f)$ being less than 10% was calculated based on 1000 randomly selected precedence list of ground motion records. Note that the $Error(s,f)$ presented in Fig. 14a corresponds to one of 1000 precedence list of ground motion records, which were randomly generated in order to determine a probability of $Error(s,f)$ being less than 10%.The results showed (Fig. 14b) that the probability of $Error(s,f)$ being less than 10% with a 90% confidence will be guaranteed, if at lease more than half of the records from the set of records are used in the analysis. It is obvious that the progressive IDA has advantage in comparison with randomly selected sub-sets of ground motion records since the $Error(s,f)$ never exceeds 10% (Fig. 13) if precedence list of ground motion records is determined by GA or simple procedure.

In order to demonstrate the ability of the progressive IDA to predict the maximum inter-storey drifts, the ratio between the maximum inter-storey drifts obtained by the progressive IDA and IDA is presented in Fig. 15. The ratio is presented for different levels of the maximum inter-storey drift up to the highly non-linear range, which is close to the collapse, and for s=4. It is obvious that the ratio varies with regard to the maximum inter-storey drift and the corresponding fractiles (16^{th}, 50^{th} and 84^{th}) and rarely exceeds 1±0.1. However, even if s=1 the ratio is in the range of 1±0.15 for most of the presented maximum inter-storey drifts. In general, the ratio is closer to one if s increases.

The important results of the progressive IDA consist of the dispersion measures which reflect record-to-record variability (randomness). The dispersion measures for randomness in S_a (β_{SaR}) as well as in drift demand (β_{DR}) can be well-predicted with the "selected" 16^{th}, 50^{th} and 84^{th} fractiles IDA curves. For example, the dispersion measures for randomness β_{SaR} and β_{DR} calculated from the results of IDA and progressive IDA in the case of s=4 are presented in Fig. 16. Note that the dispersion measures were calculated as the average value of the $\beta_{16}=\log(y_{50}/y_{16})$ and $\beta_{84}=\log(y_{84}/y_{50})$, where y_{16}, y_{50}, y_{84} represent the counted 16^{th}, 50^{th} and 84^{th} fractiles in terms of drift demand at a given spectral acceleration or vice versa.

CONCLUSIONS

Progressive incremental dynamic analysis involves a precedence list of ground motion records, which makes it possible to calculate the IDA curves progressively, starting from the first ground motion in the precedence list. When the desired tolerance in the prediction of the summarized IDA curves is achieved, the analysis can be terminated, although the IDA curves are computed only for a certain number of ground motion records from a set. This approach can significantly reduce the computational effort in the prediction of the summarized IDA curves, and thus facilitate the practical application of the IDA analysis. On the other hand, progressive IDA can be used for the selection of ground motion records from a very large set of records, provided that all records in a set are good representatives of the seismic scenario for a given site.

The case study has proven that the summarized IDA curves can be predicted with a high level of confidence when using the first fifteen records out of the ninety-eight from the precedence list. A fairly accurate prediction of the summarized IDA curves can be obtained even with only the first six ground motion records from the precedence list. The collapse capacity using only 15 ground motion records is predicted with about a 10% error, but the computational effort are reduced by more than 80%. Although the precedence lists of ground motion records, determined by a GA optimization technique and by a simple procedure, significantly differ, the results in terms of the summarized IDA curves are practically the same. Both of the optimization techniques employed in the study are appropriate for determination of the precedence list of ground motion records.

The proposed method has some limitations in this stage of the study. The precedence list of ground motion records is as good as the capability of the simple model (e.g. SDOF model) to predict the response of the structural model. The proposed method therefore may not be so effective for structures with significant higher mode effects, or for structures which can collapse in many different ways. For these types of structures the possibilities of different simple models, which are not SDOF models, have to be further investigated. In addition, further studies are needed in order to determine the precedence list of ground motion records, which is also constrained by the seismic scenario of a given site. For the presented example it was assumed that all ground motion records in a set are good representatives for the given seismic scenario, and that the scaling of the record is "legitimate".

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REFERENCES

Azarbakht, A., Dolšek, M. (2007). "Prediction of the median IDA curve by employing a limited number of ground motion records." *Earthquake Eng. Struct. Dyn.*, 36: 2401-2421.

Baker, J.W., Cornell, C.A. (2006). "Spectral shape, epsilon and record selection." *Earthquake Eng. Struct. Dyn.*, 35: 1077-1095.

Campbell, K.W. and Bozorgnia, Y. (2007). "Campbell-Bozorgnia NGA Ground Motion Relations for the Geometric Mean Horizontal Component of Peak and Spectral Ground Motion Parameters." PEER Report 2007/02, Pacific Engineering Research Center, University of California, Berkeley, California.

CEN. (2004). "Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings, EN 1992-1-1." European Committee for Standardisation, Brussels, December 2004.

CEN. (2004). "Eurocode 8: Design of structures for earthquake resistance. Part 1: General rules, seismic actions and rules for buildings, EN 1998-1." CEN, Brussels, December 2004.

Cornell, C.A., Jalayar, F., Hamburger, R.O., Foutch, D.A. (2002). "Probabilistic basis for 2000 SAC Federal Emergency Management Agency Steel Moment Frame Guidelines." *J. Struct. Eng.*, 128(4), 526-533.

Dolšek, M., Fajfar, P. (2005). "Simplified non-linear seismic analysis of infilled reinforced concrete frames." *Earthquake Eng. Struct. Dyn.*, 34(1) 49-66.

Fajfar, P. (2000). "A nonlinear analysis method for performance-based seismic design." *Earthquake Spectra*, 16(3), 573-592.

Fajfar, P., Dolšek, M., Marušić, D., Stratan, A. (2006). "Pre- and post-test mathematical modelling of a plan-asymmetric reinforced concrete frame building." *Earthquake Eng. Struct. Dyn.*, 35, 1359-1379.

Fardis, M. N. (ed.) (1996). “Experimental and numerical investigations on the seismic response of RC infilled frames and recommendations for code provisions.” *ECOEST/PREC 8 Rep. No. 6*, LNEC, Lisbon.

Foley, C. M., Pezeshk, S., and Alimoradi, A. (2007). “Probabilistic performance-based optimal design of steel moment resisting frames. I: Formulation.” *J. Struct. Eng.*, 133(6), 757–766.

Han, S.W., Chopra, A.K. (2006). “Approximate incremental dynamic analysis using the modal pushover analysis procedure.” *Earthquake Eng. Struct. Dyn.,* 35:1853-1873.

Haselton, C. B. (2009). “Evaluation of ground motion selection and modification methods: predicting median interstory drift response of buildings.” PEER Rep. No. 2009/01, Pacific Engineering Research Center, University of California, Berkeley, California.

Iervolino, I., Cornell, C.A. (2005). “Record selection for nonlinear seismic analysis of structures.” *Earthquake Spectra,* 21(3), 685-713.

Liao, K.-W., Wen, Y.-K., and Foutch, D. A. (2007). “Evaluation of 3D steel moment frames under earthquake excitations. I: Modeling.” *J. Struct. Eng.*, 133(3), 462–470.

Luco, N., Bazzurro, P. (2007). “Does amplitude scaling of ground motion records result in biased nonlinear structural drift responses?” *Earthquake Eng. Struct. Dyn.*, 36: 1813-1835.

Luco, N., Cornell, C.A. (2007). “Structure-specific scalar intensity measures for near-source and ordinary earthquake ground motions.” *Earthquake Spectra*, 23(2), 357-392.

MATLAB. (2004). The language of Technical Computing, version 7 (R14). http://mathworks.com (May 2004).

McKenna, F., Fenves, G. L., and Scott, M.H. (2000). “An object-oriented software for earthquake engineering simulation.” Univ. of California, Berkeley, California, ⟨http://opensees.berkeley.edu/⟩.

Negro, P., Pinto, A.V., Verzeletti, G., Magonette, G.E. (1996). “PsD test on four-story R/C building designed according to Eurocodes.” *J. Struct. Eng.*, 122, 1409-1417.

Negro, P., Verzeletti, G. (1996). “Effect of infills on the global behaviour of R/C frames: energy considerations from pseudodynamic tests.” *Earthquake Eng. Struct. Dyn.*, 25, 753-773.

PEER. (2005). Strong Motion Database. Available from: http://peer.berkeley.edu/nga

Shome, N., Cornell, C.A., Bazzurro, P., Carballo, E. (1998). “Earthquakes, records, and nonlinear responses.” *Earthquake Spectra*, 14(3), 469-500.

Tagawa, H., MacRae, G., Lowes, L. (2008). “Probabilistic evaluation of seismic performance of 3-story 3D one- and two-way steel moment-frame structures.” *Earthquake Eng. Struct. Dyn.*, 37:681-696.

Tothong, P., Luco, N. (2007). “Probabilistic seismic demand analysis using advanced ground motion intensity measures.” *Earthquake Eng. Struct. Dyn.* 36(13), 1837-1860.

Vamvatsikos, D., and Cornell, C. A. (2002). “Incremental dynamic analysis analysis.” *Earthquake Eng. Struct. Dyn.*, 31(3), 491–514.

Vamvatsikos, D., Cornell, C.A. (2005a). “Direct estimation of the seismic demand and capacity of multi-degree-of-freedom systems through incremental dynamic analysis of single degree of freedom approximation.” *J. Struct. Eng.*, 131(4), 589-599.

Vamvatsikos, D., Cornell, C.A. (2005b). “Developing efficient scalar and vector intensity measures for IDA capacity estimation by incorporating elastic spectral shape information.” *Earthquake Eng. Struct. Dyn.* 34(13), 1573-1600.

Villaverde, R. (2007). “Methods to Assess the Seismic Collapse Capacity of Building Strucutres: State of the Art.” *J. Struct. Eng.*, 133(1), 57–66.

Zareian, F., Krawinkler, H. (2007). “Assessment of probability of collapse and design for collapse safety.” *Earthquake Eng. Struct. Dyn.*, 36, 1901–1914.

Figure Captions

Fig. 1. Comparison between IDA analysis for a set of ground motion records and progressive IDA analysis.

Fig. 2. The schematic definition of a) *Error(s,f)* and b) *Tolerance(s,f)*.

Fig. 3. The elevation, plan view and the typical reinforcement in the beams and columns of the test structure.

Fig. 4. The elastic response spectrum (5% damping) for the ninety-eight ground motion records.

Fig. 5. The pushover curves for the positive and negative directions of loading (Fig. 3) for the MDOF model, and the idealized force-displacement relationship used for determination of the SDOF model.

Fig. 6. The single IDA curves and the summarized IDA curves (counted 16^{th} , 50^{th} and 84^{th} fractiles) computed for the simple (SDOF) model and for all the ground motion records.

Fig. 7. The *Error*(*s*,*f*) (Eq. (1)) for the precedence lists of ground motion records determined based on the simple procedure and the GA optimization technique.

Fig. 8. The *Tolerance*(*s*,*f*) (Eq. (3)) for the precedence lists of ground motion records determined based on the simple procedure and the GA optimization technique.

Fig. 9. The "selected" summarized IDA curves based on the first two subsets of ground motion records from the precedence list, which is determined with the simple procedure, the "original" summarized IDA curves, and the IDA curves for all the ground motion records.

Fig. 10. The *Tolerance*(*s*,*f*) for the progressive IDA analysis calculated for the MDOF model, using both precedence lists of ground motion records.

Fig. 11. The IDA curves, the “original” summarized IDA curves, the “selected” IDA curves for the first two (s=2) and four (s=4) subsets of ground motion records from the precedence list determined by the GA optimization technique, and the corresponding “selected” summarized IDA curves.

Fig. 12. The IDA curves, the “original” summarized IDA curves, the “selected” IDA curves for the first two (s=2) and four (s=4) subsets of ground motion records from the precedence list determined by the simple optimization technique, and the corresponding “selected” summarized IDA curves.

Fig. 13. The $Error(s,f)$ (Eq. (1)) for progressive IDA analysis for MDOF model calculated on the basis of both precedence lists of ground motion records.

Fig. 14. a) the $Error(s,f)$ (Eq.(1)) for randomly selected precedence list of ground motion records and b) the probability of $Error(s,f)$ being less than 10% versus different number of records from the precedence list. Note that the probability is determined based on 1000 randomly generated precedence list of ground motion records.

Fig. 15. The ratio between the maximum inter-storey drift (δ) obtained by the progressive IDA and IDA. The results are presented for 16th, 50th and 84th fractile (f) IDA curves and for s=4.

Fig. 16. The dispersion measures for randomness β_{SaR} and β_{DR} calculated from results of IDA and progressive IDA in the case of s=4.